

TILOTTAMA SECONDARY SCHOOL
Assignment -2077

Sub : Mathematics
Class : 12

Time : 3 Hours
F.M : 100



Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Group "A"

Attempt **ALL** the questions

1. a) In how many ways can 3 letters be posted in 4 letter boxes? [2]
b) Show that $\ln 2 = \frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \dots$ [2]
c) Prove that the identity element in a group is unique. [2]
2. a) Find the eccentricity of the hyperbola $5x^2 - 20y^2 - 20x = 0$ [2]
b) Find the ratio in which the line joining the points (2, 4, 1) and (3,5,-4) is divided by xy-plane. [2]
c) If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then prove that \vec{a} is perpendicular to \vec{b} . [2]
3. a) Using L Hospital's rule, evaluate: $\lim_{x \rightarrow 0} \frac{\tan x - x}{x - \sin x}$ [2]
b) Prove that $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \frac{a+x}{a-x} + c$ [2]
c) Solve: $\tan x dy + \tan y dy = 0$. [2]
4. a) Find the correlation coefficient between two variables having two regression coefficients -0.24 and -3.25 (if possible). [2]
b) Given : $P(A) = 0.4, P(A \cup B) = 0.56, P(B) = 0.3$ Are A and B independent events? [2]
c) What do you mean by linearly independent and dependent vectors. [2]
5. a) State the additive and multiplicative principle of counting. In an examination, a candidate has to pass in each of the 4 subjects. In how many ways can he fail? [4]

Or,

In how many ways can the letters of the word "NEPALI" be arranged so that :

- i) all the vowels are always together?
 - ii) the relative positions of vowels and consonants are not changed.
- b) Define group with an example. Let (G, *) be a group and a, b ∈ G. Then prove that a * x = b has a unique solution in G. [4]

Or,

Let $G = \mathbb{Q} - \{1\}$, the set of all rational numbers without 1. Let $*$ be an operation defined on G by $a * b = a + b - ab$. Prove that $(G, *)$ is a group.

6. a) Find the equation of the plane through the intersection of the planes $x + y + z = 6$ and $2x + 3y + 4z + 5 = 0$ and perpendicular to the plane $4x + 5y - 3z = 8$. [4]

Or,

Show that the angle between two diagonals of cube is $\cos^{-1}\left(\frac{1}{3}\right)$

- b) Show that the middle term of the expansion of $\left(x - \frac{1}{x}\right)^{2n}$ is $\frac{1.3.5 \dots (2n-1)}{n!} (-2)^n$ [4]

7. a) State Lagrange's mean value theorem. Using it for the function $f(x) = \sin x$ in $[0, x]$, prove that $\sin x \leq x$ [4]

- b) Evaluate: $\int \frac{dx}{1-3 \sin x}$ [4]

8. a) Solve the differential equation: $\frac{dy}{dx} = \frac{y}{x} - \sin^2 \frac{y}{x}$ [4]

- b) State and prove the theorem of total probability. [4]

9. Define conic section. When does a conic section become parabola? Find the equation of the parabola in its standard form. [6]

Or,

If a normal chord of a parabola $y^2 = 4ax$ subtends a right angle at the vertex, show that it is inclined at an angle $\tan^{-1} \sqrt{2}$ to the axis.

10. Define scalar product of two vectors. Give its geometrical interpretation. Prove vectorially that $b^2 = c^2 + a^2 - 2ca \cos B$. [6]

11. Following are marks secured by Mr. A and Mr. B in 10 tests of 50 marks of each.

Marks Secured by A	24	37	27	30	31	34	36	26	29	33
Marks secured by B	22	40	38	24	26	36	34	28	30	27

Who is more consistent? [6]

Group: B

- 12 a) Find the resultant and the angle subtended by it with P when the forces P and Q act at right angle. [2]

- b) A ball is thrown vertically upwards with a velocity of 30m/s. Find the time taken by the ball to reach the ground again. ($g = 10 \text{ m/ S}^2$) [2]

- c) Calculate the power of a pump which can lift 300kg of water through a vertical height of 4m in 10 sec. ($g = 10 \text{ m/ S}^2$) [2]

13.a) State and prove " Triangle of forces". [4]

b) Two points move in the same straight line starting at the same moment from the same point in it ; the first move with constant velocity u and second from the rest with constant acceleration f ; during the time that elapses before the second catches the first. Show that the greatest distance between the points is $\frac{u^2}{2f}$ at the end of time $\frac{u}{f}$ from the rest. [4]

Or,

A stone is dropped from the top of a tower 200 m. high and at the same time another is projected vertically upwards from the ground with a velocity of 50 m/s. Find where and when the two will meet? ($g = 9.8 \text{ m / s}^2$)

14 Find the resultant if two like parallel force acting on a rigid body. [6]

Or

Two like parallel forces of magnitudes P and Q are acting at the end points A and B of a rod AB of length l . If two opposite forces each of magnitude S are added to P and Q , then prove that the line of action of the new resultant will be displaced through a distance $\frac{Sl}{P+Q}$.

15. Define work, power and energy. A particle is dropped from a height h . At what height is its kinetic energy equal to $\frac{1}{4}$ of the potential energy? [6]

Group: C

16 a) Shade the feasible region under the constraints

$$y-x \geq 1, \quad y - x \leq 3, \quad 2 \leq x \leq 5 \quad [2]$$

b) Convert the hexadecimal number $(A1F)_{16}$ to octal form. [2]

c) Define diagonally dominant system of linear equations with an example. [2]

17. a) Using bisection method, find the cube root of 17 correct to within two places of decimals. [4]

Or,

Using Newtons- Raphson method, find a root of $x^3-x-4 = 0$ correct to three places of decimals.

b) Define consistency of the system of linear equations. Test the consistency of the system.

$$x + y + z = -3, \quad 3x + y - 2z = -2, \quad 2x + 4y + 7z = 7. \quad [4]$$

18. Using Simplex method, maximize $Z = 5x+3y$ subject to $2x + y \leq 40, \quad x + 2y \leq 50$ and $x, y \geq 0$ [6]

19. Given $I = \int_1^5 x^4 dx$

i) Estimate the value of I using trapezoidal rule with 5 partition points.

ii) Find the absolute error.

iii) Is the absolute error within the error bound?

[6]

Or,

How many partition points must be considered to have the approximated value of $\int_1^2 \frac{2}{x} dx$ within the accuracy of 10^{-4} ? Use Simpson's $\frac{1}{3}$ rule.

Best of Hard Labour