Class:- 11 — Quarantine Assessment — F.M = 100
Subjects:- Maths — P.M = 40

Group-A [5 × 3 × 2 = 30]

1. a) Define tautology and contradiction. Prepare the truth table of $p \vee \sim(p \wedge q)$.

b) If $x = \log_a bc$, $y = \log_b ca$, $z = \log_c ab$, prove that:

$$\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z} = 1$$

c) Examine the symmetry and odd or even nature of the function $f(x) = x - x^3$.

2 a) Prove that: $\tan^{-1}a - \tan^{-1}c = \tan^{-1}\frac{a-b}{1+ab} + \tan^{-1}\frac{b-c}{1+bc}$

b) Prove by mathematical induction that n^2+n is an even number.

c) Define transpose of a matrix. Prove that the matrices $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$ are inverse of each other.

3 a) Solve the following equation by using row-equivalent matrix method:

$2x - 3y + 6 = 0$; $3x + 2y + 9 = 0$

b) If $\alpha = \frac{1}{2}(-1+\sqrt{-3})$, $\beta = \frac{1}{2}(-1-\sqrt{-3})$, show that

$$\alpha^4 + \alpha^2\beta^2 + \beta^4 = 0$$

c) Form the quadratic equation whose one root is $\frac{1}{2+3i}$.

4 a) Find the distance between the two parallel lines $3x + 5y = 11$ and $3x + 5y = -23$

b) Find the equation of circle with centre at (4, 5) and $3x - 4y + 5 = 0$ as the line tangent to the circle.

c) Evaluate: $\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{n^2}$

5 a) Find $\frac{dy}{dx}$ of $x^3 + y^3 = 3axy$

b) Find where the graph of the function $f(x) = 2x^3 - 6x^2 + 5$ is concave downward.

c) Evaluate:- $\int \frac{1}{\sqrt{2x+1} - \sqrt{2x-3}}\, dx$

Group-B [5 × 2 × 4 = 40]

6 a) Let A, B and C be the subsets of a universal set U. Then prove the followings:

i) $\overline{A \cup B} = \bar{A} \cap \bar{B}$

ii) $A \cap (B - C) = (A \cap B) - (A \cap C)$

OR

Prove that: $\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

b) Using different characteristics, sketch the graph of $y = \cos x$ $\left(-\frac{\pi}{2} \le x \le \frac{\pi}{2}\right)$

7 a) If $\cot^{-1}x + \cot^{-1}y + \cot^{-1}z = \pi$, show that $xy + yx + zx = 1$

OR

In a ΔABC, if $(\sin A + \sin B + \sin C)(\sin A + \sin B - \sin C) = 3 \sin A \sin B$ then prove that $\angle C = 60°$.

b) By using the properties of determinant, show that:

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

8 a) Apply row-equivalent matrix method or matrix inversion method to solve the following system of linear equations
$2x - y + 4z = -3$, $x - 4z = 5$, $6x - y + 2z = 10$

b) Find the condition for the two given quadratic equations $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ may have one root common and both roots common.

9 a) Find the equation of the circle which passes through the centre of the circle $x^2+y^2+8x+10y-7=0$ and is concentric with the circle $2x^2+2y^2-8x-12y-9=0$.

b) Evaluate $\lim_{x\to\theta} \frac{x\cos\theta - \theta\cos x}{x-\theta}$

OR

When a function $f(x)$ is said to be continuous at a given point $x=a$? Discuss the continuity of the function

$$f(x) = \begin{cases} \frac{x^2-x-6}{x^2-2x-3}, & x \neq 3 \\ \frac{2}{3}, & x=3 \end{cases} \quad \text{at } x=3.$$

10 a) Find, from definition the derivative of $y=\log_5 x$.

b) Using integration, find the area of region between $y^2=4ax$ and $x^2=4ay$.

Group-'C' [6×5 = 30]

11. Show that $f:R\to R$ defined by $f(x)=3x+4$ is one-to-one and onto. Find $f^{-1}(x)$. Also prove that $fof^{-1}(x) = f^{-1}of(x) = x$

12. Define arithmetico-geometric series. Sum to n terms the series: $\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{16}+\cdots$

13. The origin is a corner of a square and two of its sides are given by $2x+y=0$ and $2x+y=3$. Find the equations of the other two sides.

OR

For what value of c, the lines which join the origin to the point of intersection of the line $x-y+c=0$ and the curve $x^2+y^2+4x-6y-36=0$

14) State De-Moivre's Theorem. Use it to find the cube roots of unity. Write their properties.

15) What are the criteria to be satisfied in order to have maximum and minimum values of a function? Find the extreme values of the function $f(x) = x^5-5x^4+5x^3-1$. Also find the point of inflection.

OR

A man 1.5m tall walks at a uniform speed of 5 km/hr away from a lamp-post 6m high. Find the rate at which the length of his shadow increases.

~ Best wishes ~